– ABSTRACT –

Euler in Two Acts

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Leonhard Euler (1707 – 1783) is one of the towering figures from the history of mathematics. Here we look at two results that show how he acquired his lofty reputation.

In the first, Euler considers the infinite series $1/2 + 1/3 + 1/5 + 1/7 + 1/11 + \ldots$ – i.e., the sum of reciprocals of the primes – and establishes that the sum "is infinite." The proof from 1737 rests upon his famous product-sum formula and requires a host of analytic manipulations so typical of Euler's work.

The other result involves $1 + 1/4 + 1/9 + 1/16 + \ldots$ – i.e., the sum of reciprocals of the squares. Euler first evaluated this in 1734, but here we examine his 1755 argument that uses l'Hospital's rule, not once, not twice, but *thrice*!

Euler has been described as "analysis incarnate." These two theorems, it is hoped, will leave no doubt that such a characterization is apt.

NOTE: This talk should be accessible to any mathematics major or minor.